EE 341
Discrete Time Linear Systems

Lab 3: The FFT and Digital Filtering

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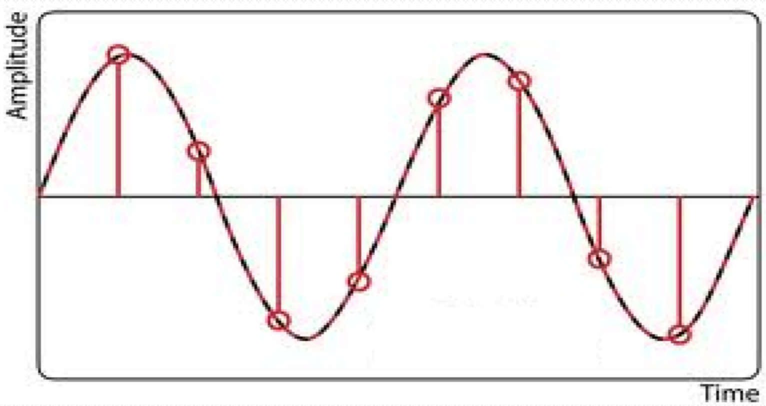
Lab 3: The FFT and Digital Filtering

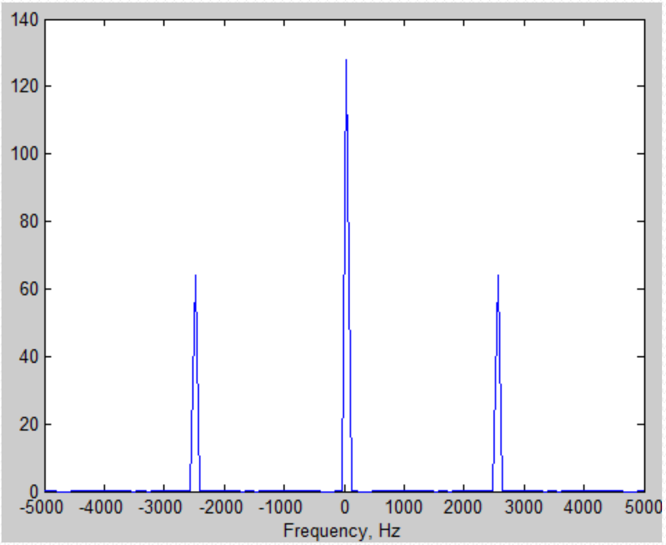
- Assignment 1
- Assignment 2
- Assignment 3
- Assignment 4
- Assignment 5

What you will learn in this lab

- Process the discrete time signal in frequency domain by using MATLAB.



Horizontal axis: time (sec)



Horizontal axis: frequency (Hz, radian)

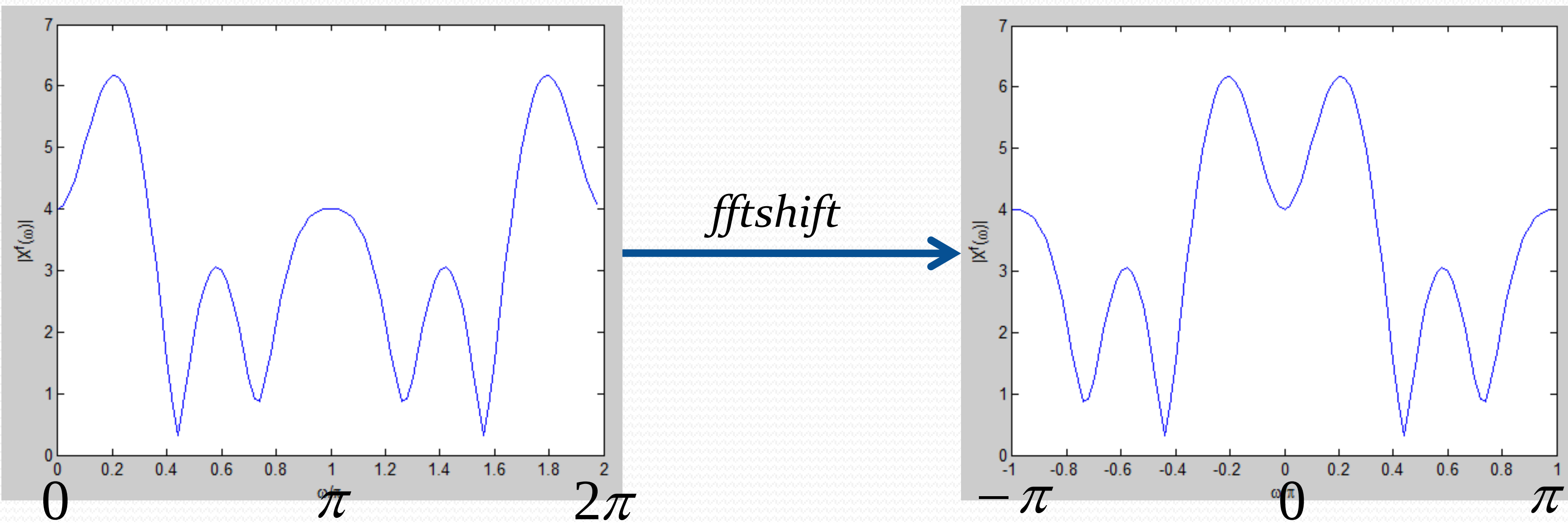
Assignment 1: MATLAB function FFT

1. MATLAB command : *fft* (Fast Fourier Transform)
   - It computes Discrete Fourier Transform (DFT) of a sequence efficiently.
     - *y = fft(x);*
     - Magnitude ➔ *abs(y)*
     - Phase ➔ *angle(y)*
   - Check "*help*" for more details

2. MATLAB command : *fftshift*
   - Change *fft* output range (in frequency)
     - *z = fftshift(y);*
   - Why do we need shift ?

Assignment 1: MATLAB function FFT

- The "*fft*" outputs a sequence in the range $0 \leq \omega \leq 2\pi$




```
w1 = linspace(0, 2*pi, N);
plot(w1, abs(y))
```

```
w2 = linspace(-pi, pi, N);
plot(w2, abs(z))
```

- But it is more natural to plot in the range $-\pi \leq \omega \leq \pi$
  - Then the DC component is in the middle of the spectrum

DFT: $X[k], \quad k = 0 \sim N-1 \quad \Rightarrow \quad \omega = \frac{2\pi}{N}k$

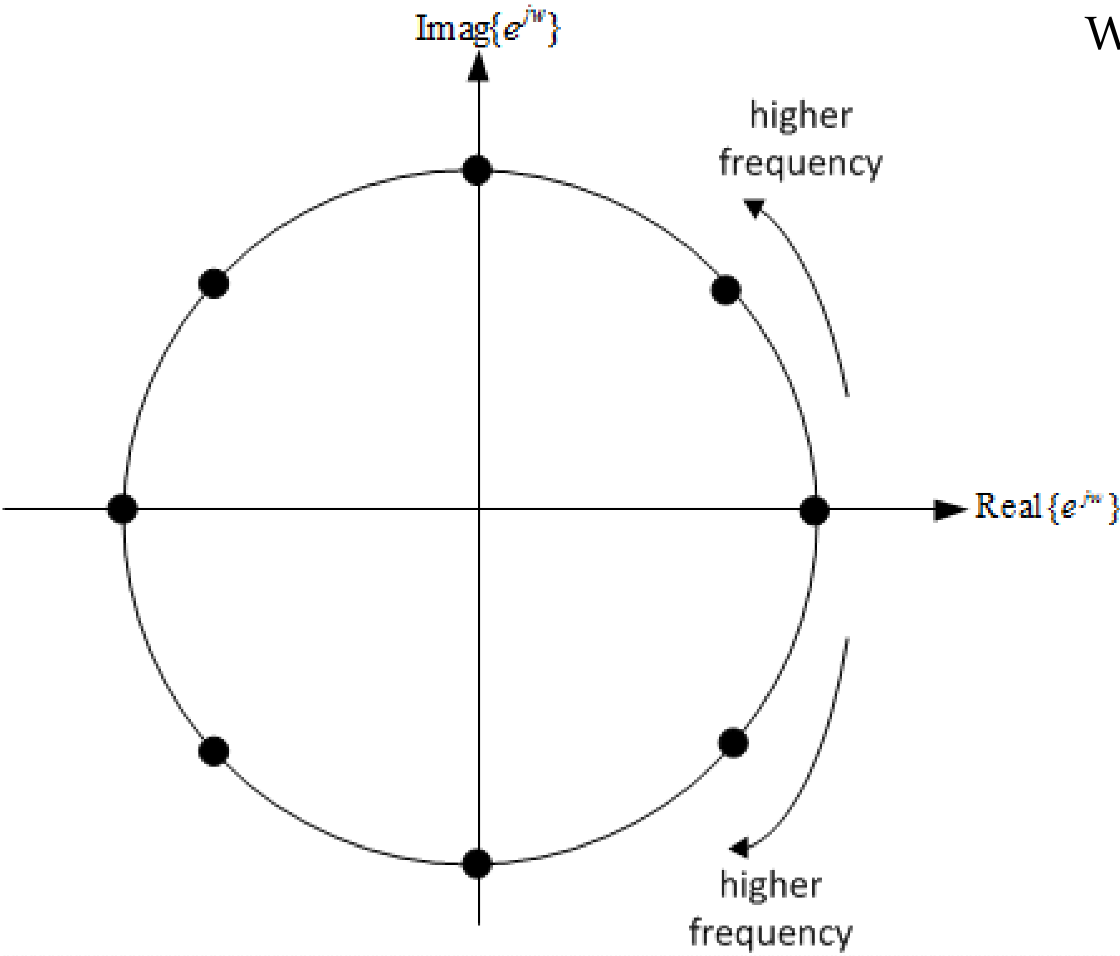
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-\frac{2\pi}{N}kn}$$

$$X[N-k] = \sum_{n=0}^{N-1} x[n] e^{-\frac{2\pi}{N}(N-k)n} = \sum_{n=0}^{N-1} x[n] e^{\frac{2\pi}{N}kn} = (X[k])^*$$

$$\Rightarrow \left|X[N-k]\right| = \left|(X[k])^*\right| = \left|X[k]\right|$$

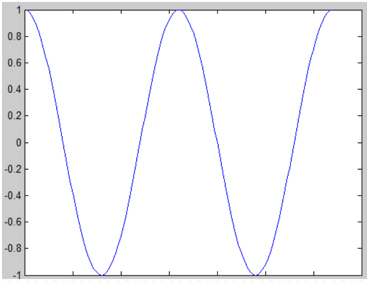
So the magnitude plot is symmetric around w=pi

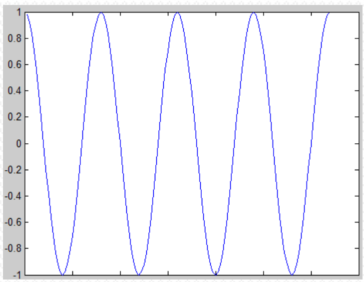
$$\textbf{DFT:}\ \ X[k],\quad k=0\sim N-1\ \Rightarrow\ \ \omega=\frac{2\pi}{N}k$$



Which one has higher frequency?

1. $\cos((\frac{2\pi}{8})n)$ $v.s.$ $\cos((\frac{2\pi}{8}3)n)$
2. $\cos((\frac{2\pi}{8})n)$ $v.s.$ $\cos((\frac{2\pi}{8}7)n)$
3. $\cos((\frac{2\pi}{8}3)n)$ $v.s.$ $\cos((\frac{2\pi}{8}7)n)$





Assignment 1: MATLAB function FFT

- Short summary
  - 1. Frequency components are symmetric with respect to $\omega = \pi$
  - 2. Highest frequency in radian of discrete time signal is π, not 2π

Assignment 1: MATLAB function FFT

- Frequency in Hz (f) and normalized frequency $(\bar{f})$
  - Given $x[n]=1+\cos(2\pi\bar{f}n)$ $\bar{f}=0.25\ or\ 0.5$, what is the frequency in Hz?
    normalized frequency

$$x(t)=1+\cos(2\pi ft) \qquad f\ is\ in\ Hz$$

$$x[n]=1+\cos(2\pi f(nT_s))=1+\cos(2\pi\bar{f}n)$$

$$=>\ \bar{f}=fT_s=\frac{f}{f_s} \qquad \text{normalized frequency}=\frac{\text{frequency in Hz}}{\text{sampling frequency}}$$

$$e.g.\ Sampling\ period\ T_s=10^{-4},\ \bar{f}=0.25\ or\ 0.5$$

$$=>\ f=\bar{f}\times 10^4=2500\ \ or\ \ 5000\ \ Hz$$

Assignment 1: MATLAB function FFT

- Normalized frequency ($\bar{f}$) and frequency in radian (ω)

$$x[n] = 1 + \cos(2\pi\bar{f}n) = 1 + \cos(\omega n)$$

$$\Rightarrow 2\pi\bar{f} = \omega$$

$$\Rightarrow 2\pi\frac{f}{f_s} = \omega$$

$$0 \le f \le f_s \quad <=======> \quad 0 \le \bar{f} \le 1 \quad <=======> \quad 0 \le \omega \le 2\pi$$

$$-\frac{f_s}{2} \le f \le \frac{f_s}{2} \quad <=======> \quad -\frac{1}{2} \le \bar{f} \le \frac{1}{2} \quad <=======> -\pi \le \omega \le \pi$$

after *fftshift()*

Summary for assignment 1

- Use f= 0.25 and 0.5
- Plot magnitude of (1) unshifted DFT, (2) shifted DFT; both in radians.

$\boxed{(0 \le w \le 2\pi)}$ $\boxed{(-\pi \le w \le \pi)}$

- Plot magnitude of (3) shifted DFT in Hz

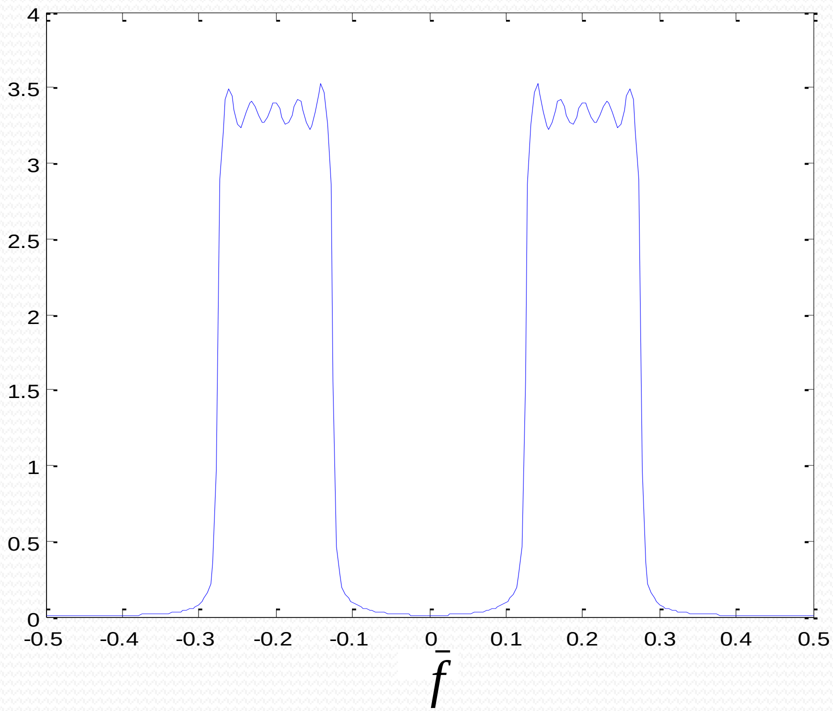
$$\boxed{(-\frac{f_s}{2} \le f \le \frac{f_s}{2})}$$

- Please indicate which one you are plotting and the frequency axis as well. You can use "subplot" to show different plots in the same window.
- Total 6 images
- Answer the questions in the lab document. (The text with underline)

Assignment 2: Frequency Shifting

- $x_3[n]=\text{sinc}(f_1(n-32))\cos(2\pi f_2 n)$, where $f_1 = 0.15$, $f_2 = 0.2$, n= 0 ~ 255
  - x3 = sinc(f1*(n-32)).*cos(2*pi*f2*n);
  - y3 = fftshift(fft(x3));
- FFT Result:

  Magnitude of y3

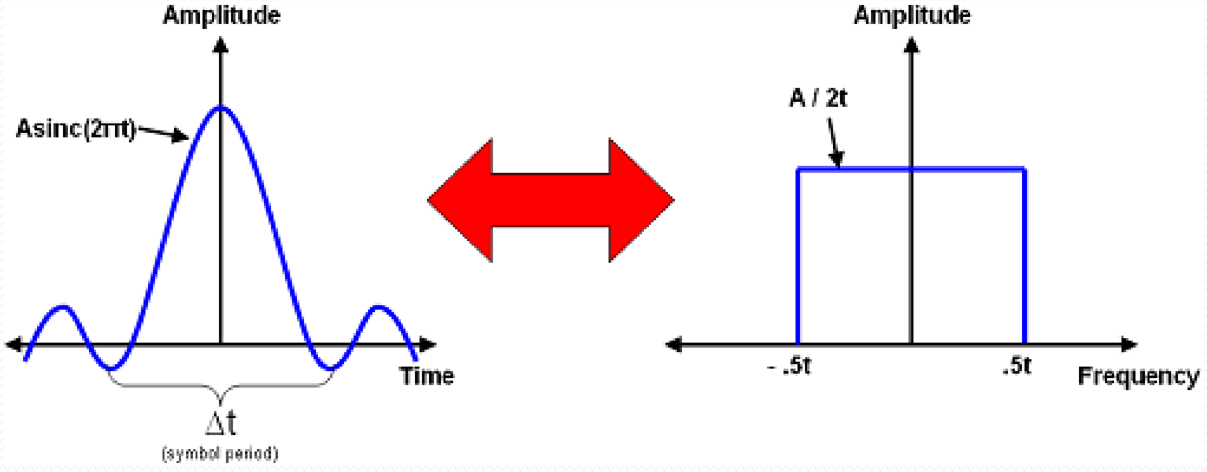


Assignment 2: Frequency Shifting

1. Important Concept : convolution property
   - Suppose e[n], g[n] are two the discrete signal
   - E[ω], G[ω] are FFT results for e[n] and g[n]

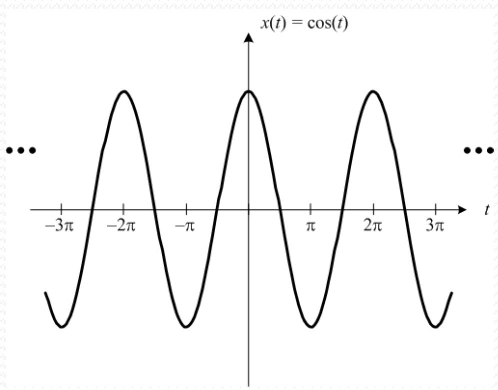
$$e[n]\cdot g[n] \xrightarrow{FFT} \frac{1}{2\pi}\mathrm{E}[\omega]\otimes \mathrm{G}[\omega]$$

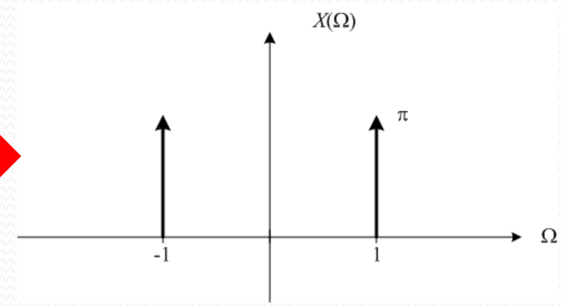
2. if e[n] is the sinc function ➔ E[ω] will be a rect. function



Assignment 2: Frequency Shifting

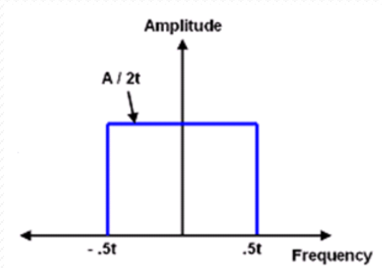
3. If g[n] is the cosine function➔ G[ω] will be impulse signal pairs





- From 1~3 above, we can know that the *fftshift*(*fft*(x3)) can be viewed as

$$(E[\omega]*\delta[\omega-M]=E[\omega-M])$$



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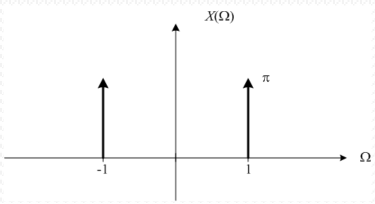
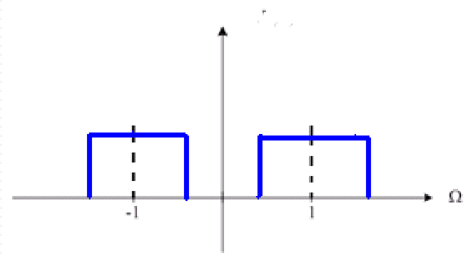




Summary for assignment 2

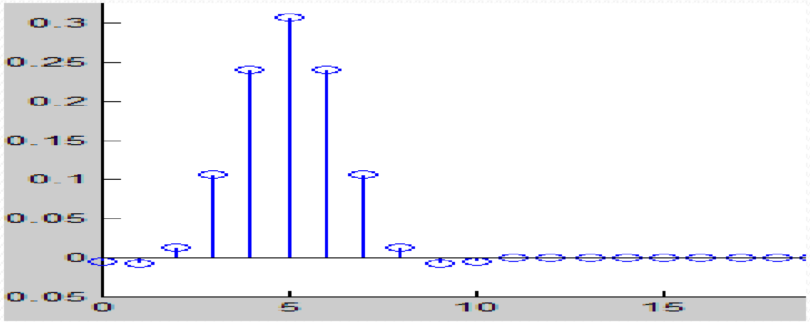
- Turn in the magnitude and phase plots only for (d) x4[n]. The frequency range is in $-0.5 \le \bar{f} \le 0.5$ Normalized frequency
- You can use "subplot" to show different plots in the same window.
- Total 2 images
- Answer the questions in the lab document. (The text with underline)

Assignment 3: FIR Digital Filters

- FIR: A filter with Finite Impulse Response
  - *b_lowfir = fir1(filter_order, cut-off_freq);*
  - The *cut-off_freq* is the frequency in radian (ω) normalized with respect to π e.g. if ω is 0.3π ; what will $\bar{\omega}$ be?

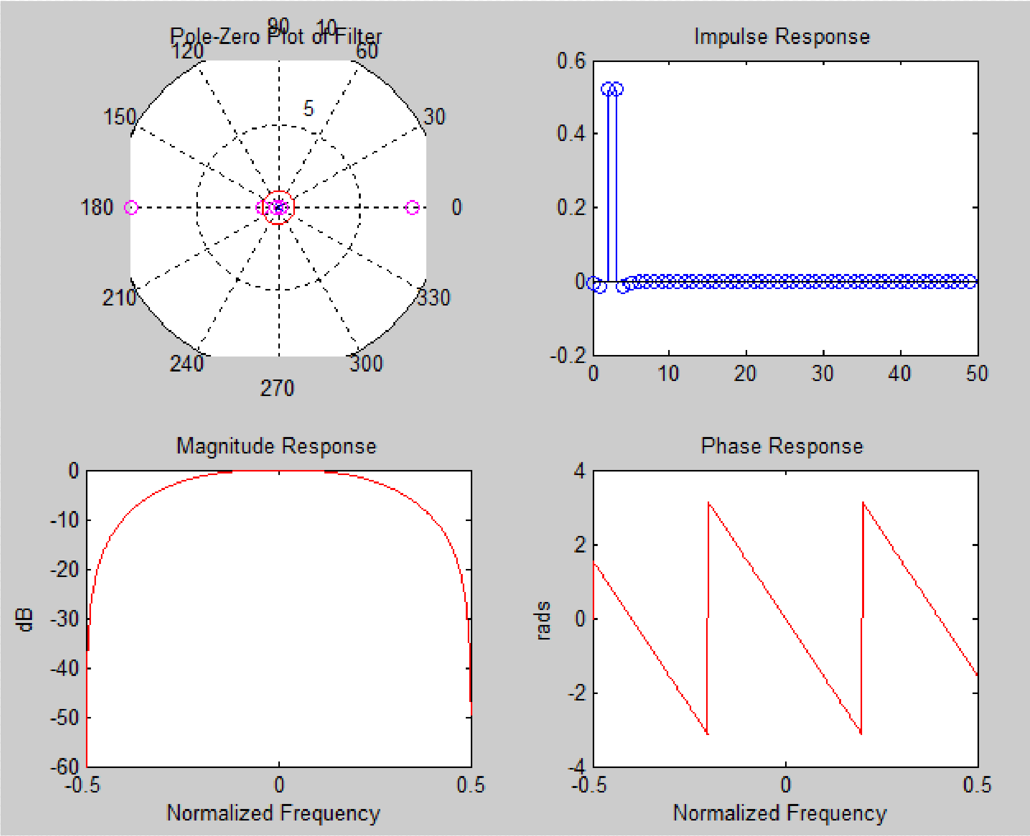
$$0 \le \bar{\omega} \le 1 <====> 0 \le \omega \le \pi$$

- Use *frevalz01()* to analyze the system
  - *frevalz01(b_lowfir , 1)*
    - Only two inputs are necessary. The 2nd input for *frevalz01* should be set as 1 here.



Assignment 3: FIR Digital Filters

- Ex: order of 5 with cutoff frequency of 0.7π
  - *b_lowfir = fir1(5, 0.7);*
  - *frevalz01(b_lowfir , 1);*

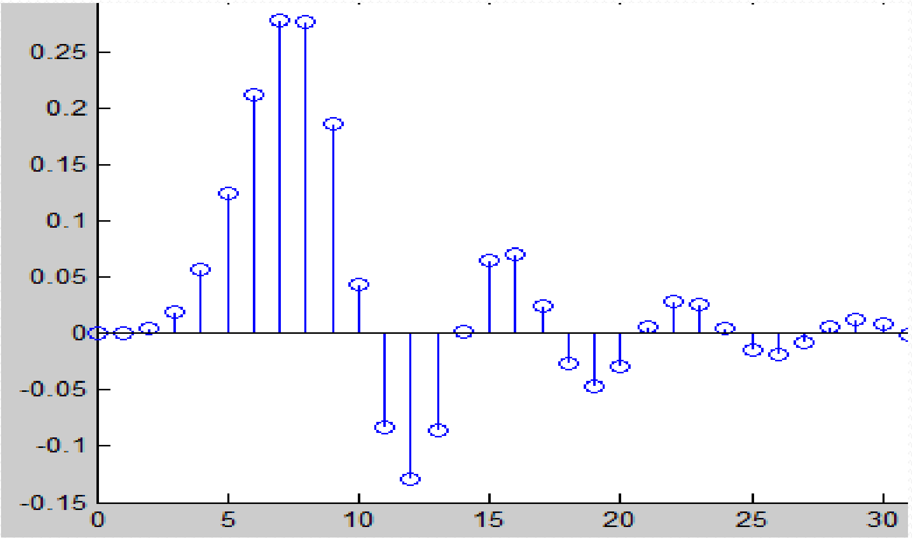


Summary for assignment 3

- Turn in the *frevalz01* plot.

Assignment 4: IIR Digital Filters

- IIR: A filter with Infinite Impulse Response
  - *[b_lowbutt, a_lowbutt] = butter(filter_order, cut-off_freq);*
  - *frevalz01(b_lowbutt , a_lowbutt);*



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Summary for assignment 4

- Turn in the *frevalz01* plot.
- Answer the questions in the lab document. (The text with underline)

Assignment 5: Filter Implementation

- Matlab Command: *filter*
  - *x[n]* is the input signal: u[n]-u[n-20] with 40 zeros appended. So the total length is 60.
  - *y_fir = filter(b_lowfir, 1, x);*
  - *y_butt = filter(b_lowbutt, a_lowbutt, x);*

Summary for assignment 5

- Turn in the plots of (1) input signal x (2) output *y_fir* after appending zeros (3) output *y_butt*
- You can use "subplot" to show different plots in the same window.
- Total 3 images
- Answer the questions in the lab document. (The text with underline)

More About Lab 3

- For assignment 1, the 2nd and 3rd plots should be almost the same. The difference between them is the horizontal axis. Because they have different units in the horizontal axis, the labeled values are different, too.
- You need to follow the requirement in the slides, such as indicating clearly and correctly the horizontal axis.
- The main reason that we want to label the horizontal axis in different units is because it is sometimes easy to observe in some specific unit.

More About Lab 3

- If you want to discuss if the peak locations make sense, you can try to find out what is the result after you transform the input signal into frequency domain.
- You can derive by your hand with the definition of Fourier transform or look up the transform pair table in the text book.
- After you get the result, you will see where should the peaks be.

More About Lab 3

- If you want to perform N point FFT to input x[n], you just add an extra input N to the command *fft*.
  - *fft*(x, N)
- In this lab, either you specify the N or you don't is fine for us. If you don't specify, it will automatically take the length of the x[n] as default N.

More About Lab 3

- (i) Please see clearly the questions mentioned in the documents. You need to answer all the questions in order to get the full credit.
- (ii) Please read the slides and document carefully. You need to follow the guidelines in the slides.
- (iv) There is no need to submit the code. You need to turn in the hard copy report in the lab session on the due date.